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Understanding the Time Value of Money

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1. INTRODUCTION

Most of the transactions concluded in the financial markets are based on the *time value of money*. The time value of money simply refers to the *value of money over a specified period of time*.

Taking time value into consideration, you would rather receive R100 today, than receive R100 one year from today. The reason for this is that you could invest the money now and receive interest for the period. In other words, there is value in holding the R100 over the one-year period. *The time value of money arises because we can invest money to earn interest over time*.

Whether one considers the short-term cash markets (money markets), bond market instruments, equities or the most sophisticated derivative instruments, these are all about *buying and selling cash flows of some nature*. But to be able to understand how we can establish a value for all those different cash flows, we have to be able to value money at different points in time.

Generally, financiers are interested in three aspects of the time value of money. These are:
- **Future value** – the value of a sum of money invested today for a given period of time
- **Present value** – today's value of a sum of money expected to be received at some point in the future
- **Interest rates** – the return one will receive on one's money over a specified period of time.

2. FUTURE VALUE

If you were to deposit an amount of R100,000 into a savings account at an interest rate of 10% per annum calculated on a monthly basis, what will the future value of this investment be after a period of three years?

To answer this question, one needs to understand what factors affect future value:
Time – the number of days, months or years the money is invested for;
Rate – the interest rate applied to the funds for the period.

The ***higher*** the interest rate, the ***greater*** the future value of a cash flow
The ***longer*** the period, the ***higher*** the future value of a cash flow

Therefore, the *actual growth* in a cash flow *over time* depends on the *way in which interest is paid*. The first step is to distinguish between *simple interest* and *compound interest*.

2.1 Simple interest

Simple interest is used to describe investments in which *interest is only paid on the initial investment or principal amount*, denoted in our equations by the letter 'P'.

Compound interest describes investments where *interest payments as well as the original principal are reinvested at the end of each interest period to earn further interest (interest-on-interest).*
As simple interest is only paid on the principal it accrues at a constant or absolute rate. The amount of interest (IA) is equal to the product of principal (P) times the interest rate (i) times the number of periods (t):

IA = P x i x t

The future value of an investment will be given by:

$$\begin{aligned} FV &= P + i \\ &= P + (P \times i \times t) \\ &= P \times [1 + (i \times t)] \end{aligned}$$

(Note that the interest rate (i) in all these equations is expressed as a decimal i.e. 5% = 0.05 and 11% = 0.11.)

EXAMPLE

An investment of R100 at a simple interest rate of 10% per annum will grow to R110 over a one year period: FV_1 = 100 x (1.10) = R110

- ***R100 is the original amount invested***
- ***R10 is interest earned over the one-year period***

Simple interest calculations are typically used when an annual interest rate is specified but the actual term of the investment is *less than one year*. For example, the interest accrued on *money market instruments* such as deposits and certificates of deposit is calculated using simple interest.

Like most financial markets, the money markets quote the interest due as an *interest rate for a full year*, which must then be modified by the time period to which it applies. Although this does create certain complexities in calculation, it does allow us to be able to *compare interest rates for different periods* very easily.

EXAMPLE

If there are 365 days in a year and the interest rate is 8% per annum then the interest accrued over 182 days on R100,000 is given by:

***i* = R100,000 x (0.08) x (182/365) = R3,989.04**

2.2 Compound Interest

As mentioned, compound interest refers to the *re-investment of interest paid* at the end of each period, so that the interest already earned receives interest in subsequent investment periods. If R100 is placed in a bank account to earn interest at a constant rate of 10%, it will grow to R110 by the end of the first year. If the R110 is then fully reinvested, it will grow to R121 by the end of the second year and to R133.10 by the end of the third year. This is set out in the table below.

Year	Sum invested at beginning of the year	Interest	Balance at the end of the year
1	**R100**	**R10**	**R110**
2	**R110**	**R11**	**R121**
3	**R121**	**R12.10**	**R133.10**

Since the interest that is earned in the first year is reinvested in the second year, it will itself earn interest. This is the basis of the compound growth process.

The calculation can be set out formally as follows:

$$FV_1 = 100 + (100 \times 0.10) = 100 \times (1 + 0.10) = 100 \times (1.10) = 110$$

where FV1 stands for the value of the investment at the end of one period and 0.10 represents the interest rate of 10% expressed in decimal form. If the money is left in the deposit account for a second year, it will grow to R121 as shown below:

$$FV_2 = 110 + (110 \times 0.10) = 110 \times (1 + 0.10) = 121$$

However, if we recognise that FV1 = 110 = 100 (1 + 0.10), this can be rewritten as:

$$\begin{aligned} FV_2 &= 100 \times (1 + 0.10) \times (1 + 0.10) = 100 \times (1 + 0.10)^2 \\ &= 100 \times (1.21) \\ &= 121 \end{aligned}$$

The interest earned during the second year is R11, of which R10 was earned on the original capital amount of R100 and R1 earned on the interest of R10 accrued in the first year of investment. On this basis, an investor should be indifferent between R100 today and R121 in two year’s time.

3. DISCOUNTING AND PRESENT VALUES

The previous sections showed that it is easy, given the *appropriate interest rate*, to calculate the *equivalent future value of some amount of money invested or borrowed today*. Suppose instead that we know what a future cash flow is going to be, and need to calculate what the *equivalent value of that cash flow* would be today. We refer to this as the *present value*.

For example, suppose that in exactly one year’s time a student will have to pay R2,000 in tuition fees, but desperately wants to avoid taking out a loan in order to pay the tuition fees.

Instead, the student decides to set aside some money in a savings account that pays interest at a rate of 6.5% per annum. What the student needs to know is:

What is the minimum amount that needs to be put into the account today in order to have sufficient funds to pay for the tuition fees in a year’s time?

The process used here is the *reverse of the compounding process* and is known as *discounting*.

Looking again at the *future value equation* derived above:

$$FV_n = P_0 \times (1 + i)^n$$

When using this equation in the previous section, we knew P_0, (the amount we were investing or borrowing today), and the rate of interest *(i)* per annum at which we were doing so. Thus, we could calculate the unknown FV_n. In our example given above, the student knows the future value of the investment is R2,000 — and that the money can be invested at 6.5%. What the student needs to know is the amount that must invested today. For the student the *unknown is* P_0.

Dividing both sides of the above equation by the *interest factor* $(1 + i)^n$ we have:

$$P_0 = \frac{FV_n}{(1+i)^n} = FV_n \frac{1}{(1+i)^n}.$$

In this form, 'i' is referred to as the *discount rate* and:

$\frac{1}{(1+i)^n} = (1+i)^{-n}$ as the *discount factor or present value factor* for period n.

Returning to our student:

$$P_0 = \frac{R2{,}000}{(1.+0.065)^1} = R2{,}000 \text{ x } \frac{1}{(1+0.065)^1} = R1{,}877.93$$

That is, the student will have to put a minimum of R1,877.93 into the savings account today in order to have sufficient funds to pay the tuition fees in one year's time. In other words, given an interest rate of 6.5% the present value of R2,000 in one year's time is R1,877.93.

The difference between R2,000 in one year's time and its present value of R1,877.93 is simply a reflection of the *time value of money*.

So far we have only considered the PV of a cash flow in the future, but in many situations a *series of future cash flows*, rather than a single payment, need to be valued.

For example, consider the present value of an instrument that pays an annual interest amount of 7% and matures in exactly five years. Here, there are four equal cash flows paid to the investor at the end of the next four years (interest) plus a final payment equal to the principal together with the final interest payment on maturity of the investment (a typical bond investment). Logically the *present value of the bond has to be the sum of the present value of each of the individual cash flows*:

$$PV = \frac{C}{(1+i)^1} + \frac{C}{(1+i)^2} + \frac{C}{(1+i)^3} + \frac{C}{(1+i)^4} + \frac{C + \text{Principal}}{(1+i)^5}$$

Where C is the *annual interest amount* and i is the *prevailing market interest rate*. More generally, the present value of a series of future payments can be written as:

$$PV = \frac{C_1}{(1+i)^{t_1}} + \frac{C_2}{(1+i)^{t_2}} + \frac{C_3}{(1+i)^{t_3}} + \dots + \frac{C_n}{(1+i)^{t_n}}$$

EXAMPLE

Applying this in practice, if an investment promises the payment of R1,000 at the end of years one and two and the prevailing interest rate is 10% per annum, the value of the investment in *today's terms* is given by:

$$\mathbf{PV} = \frac{\mathbf{1{,}000}}{(\mathbf{1}+\mathbf{0.1})^{\mathbf{1}}} + \frac{\mathbf{1{,}000}}{(\mathbf{1}+\mathbf{0.1})^{\mathbf{2}}}$$

$$= (\mathbf{1{,}000} \times \mathbf{0.90909}) + (\mathbf{1{,}000} \times \mathbf{0.82645})$$

$$= \mathbf{R1{,}735.54}$$

4. PERIODIC PAYMENTS OR ANNUITIES

An *annuity* is the term used to describe *a series of level (equal) payments over time made on a regular basis* e.g. monthly, quarterly, annually etc. There are three main types of annuities:

Ordinary Annuity: a series of *equally spaced payments (or receipts) of the same amount over a determined period*, the first payment being *due at the end of the first sub-period*. This would cover, for example, interest payments made on a fixed rate bond.

Annuity Due: a series of equally spaced payments (or receipts) of the same amount over a determined period, the *first payment being due at the beginning of the first sub-period*. This type of annuity is more usually found in the *insurance* world and would be very rare in the debt markets.

Perpetuity: an annuity that *extends forever into the future*.

This document only considers the valuation of an ordinary annuity for introduction purposes.

4.1 Future value of an annuity

If we were to invest R1 on 31 December of each year to earn interest at 5% per annum, what *terminal (maturity) amount* would we have at the end of 10 years? The initial investment at the end of the first year would earn interest on a compound basis for nine years, the investment at the end of the second year would earn interest on a compound basis for eight years and so on. The investment at the end of year 10 would, of course, earn no interest.

To compute the value, we could calculate the future value of each investment individually:

$$\mathbf{FV = [1 \times (1.05)^{9]} + [1 \times (1.05)^{8}] + [1 \times (1.05)^{7}] + \ ... \ + \ [1 \times (1.05)^{1}] + 1}$$

4.2 The present value of an ordinary annuity

Just as we derived a formula to calculate the *future value of an annuity*, we can also find one to work out the *present value of a stream of payments*.

If an investor expects to receive R1 on 31 December each year for 10 years, and the interest rate is 5% per annum, what is the present value of the stream of payments on 1 January, 12 months prior to the first payment?

Clearly the answer could be obtained by finding the PV of each individual payment:

$$PV = [1 \times (1.05)^{-1}] + [1 \times (1.05)^{-2}] + [1 \times (1.05)^{-3}] + \ldots + [1 \times (1.05)^{-10}]$$

5. TIME VALUE OF MONEY FUNCTIONS IN EXCEL

Spreadsheets are used very widely and are a natural choice when evaluating cash flows and financial instruments. Excel has a number of financial functions built into it and this section reviews the application of those that relate to the time value of money. Of course, it is very easy to set up a spreadsheet to find PVs and FVs using the basic formulas set out in this workbook, but we want to present here the formal functions built into the application. Note that a number of these might need to be loaded as add-ins to the basic programme but should be accessible from the programme disk or your technical support if you cannot find them listed in the 'Insert Function' menu.

FV = (rate, nper, pmt, pv, type)
Returns the future value of an investment based on periodic, constant payment and a constant interest rate.

PV = (rate, nper, pmt, fv, type)
Returns the present value of an investment: the total amount that a series of future payments is worth now.

NPV = (rate, value 1, value 2, …)
Returns the net present value of an investment based on a discount rate and a series of future payments (negative values) and income (positive values).

EFFECT = (nominal_rate, npery)
Returns the effective annual interest rate.

NOMINAL = (effect_rate, npery)
Returns the annual nominal interest rate.

IRR = (values, guess)
Returns the internal rate of return for a series of cash flows.

MIRR = (values, finance_rate, reinvest_rate)
Returns the internal rate of return for a series of periodic cash flows, considering both cost of investment and interest on reinvestment of cash. One of the problems in using the standard internal rate of return calculation is that it assumes reinvestment of cash flows at the IRR, which may not be realistic or how we wish to do the analysis. This function allows the user to define a different reinvestment rate.

PMT = (rate, nper, pv, fv, type)
Calculates the payment for a loan based on constant payments and a constant interest rate — calculating an annuity amount (PMT = payment).

YIELD = (sett, mat, rate, pr, redemption, frequency, basis)
Returns the yield on a security that pays periodic interest (sett = settlement; mat = maturity). Basis refers to the interest convention for the particular security.

6. KEY POINTS SUMMARY

- As money can earn interest, it has different values at different points in time.
- We cannot therefore compare or add money stated at different points in time without adjustment.
- We can work out the future value of money today, or the present value of an amount of money in the future, by using the appropriate interest rate to compound or discount between the future and now. Once we have money on a consistent time basis we can then add or compare values.
- In making this calculation over a number of periods, we implicitly compound the value by the interest rate used. This is not realistic, but as we cannot foretell the future we must live with it and recognise its limitations.
- We can use these techniques to value a range of financial instruments: bonds, annuities, etc. Although we must recognise this will give us their theoretical, fair value rather than their market value which will be affected by supply and demand.
- The frequency with which interest is paid will affect the effective return/cost. Always ensure that you are comparing interest rates on a like basis and convert if necessary.
- The calculations in this document basically demonstrates that time is literally money — i.e. that the value of money you have now is not the same as it will be in the future and vice versa.

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